Advanced Algorithms

September 11, 2025

Logistics

- Pick up Exercise solutions.
- Assignment 1 is released on the course webpage
- Due Tuesday 23rd on **gradescope**. Make sure you are in the gradescope. Two free late days. Do not do at the last minute.
- Collaboration is encouraged. The problems are not straightforward. I've put steps and guidance. Come to me if stuck.
- Download it again if you already did so earlier (I took out problem 2 part f)

Exercises

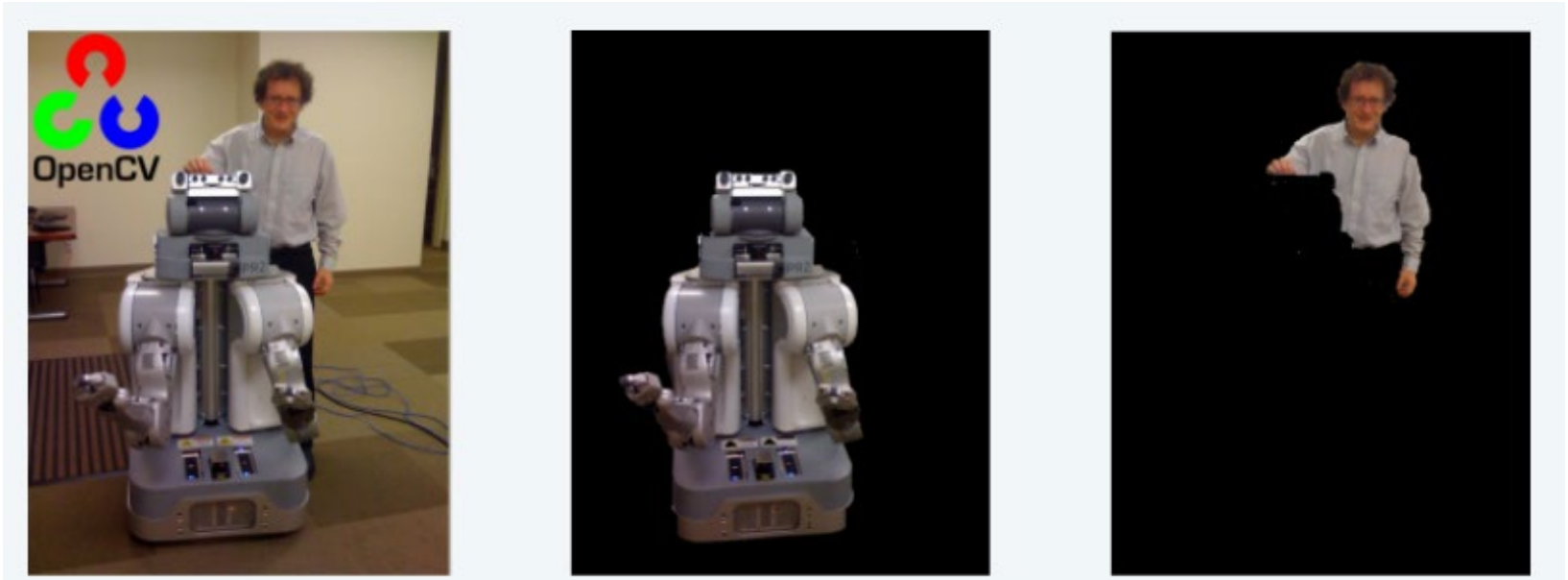
- What would you like me to go over from this week’s exercise set?

Algorithms and Applications

Image Segmentation

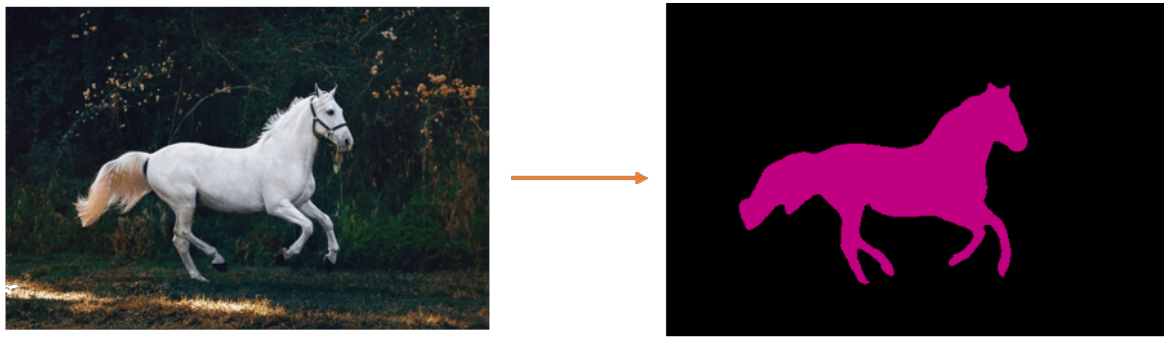
Image segmentation: separate the pixels of an image into distinct shapes corresponding to contiguous figures

Central problem in image processing



Foreground / background segmentation

As a start: distinguish the foreground from the background



Problem formulation

- Form groups of 3-4

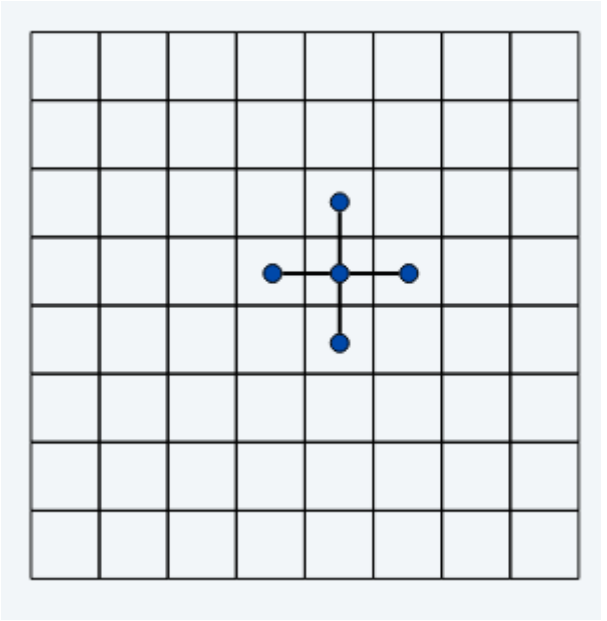
- Fuzzy problem: Given an image, decide which pixels are in the foreground and which are in the background

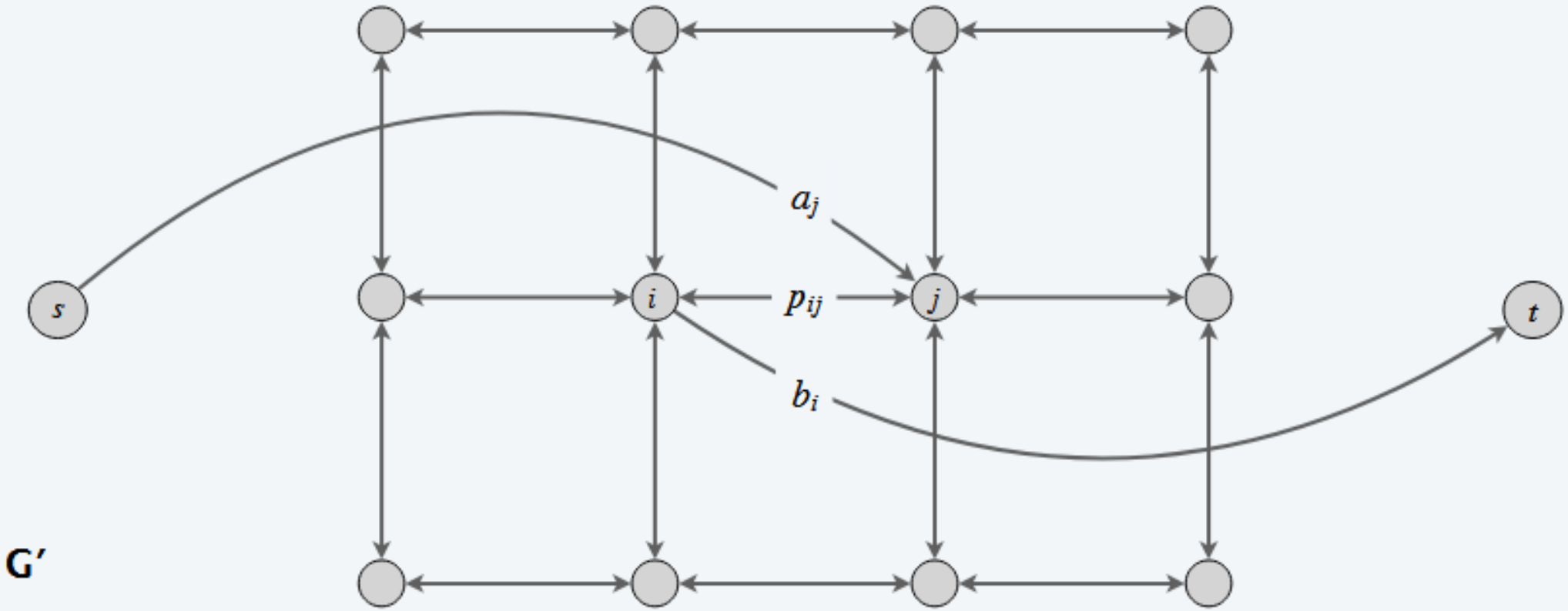
- Your task: model this as an optimization problem
  - What is the input to your problem? I.e. what do you need from the image
  - What is a feasible solution?
  - How to compare feasible solutions?

Themes?

An approach

- We will consider each pixel in the image as a vertex.
- Each pixel (node) is adjacent to its “neighbors”
- These could be the pixels immediately to the left, right, up and down of it.
- Confidence of foreground / background given, as well as penalty for separation.

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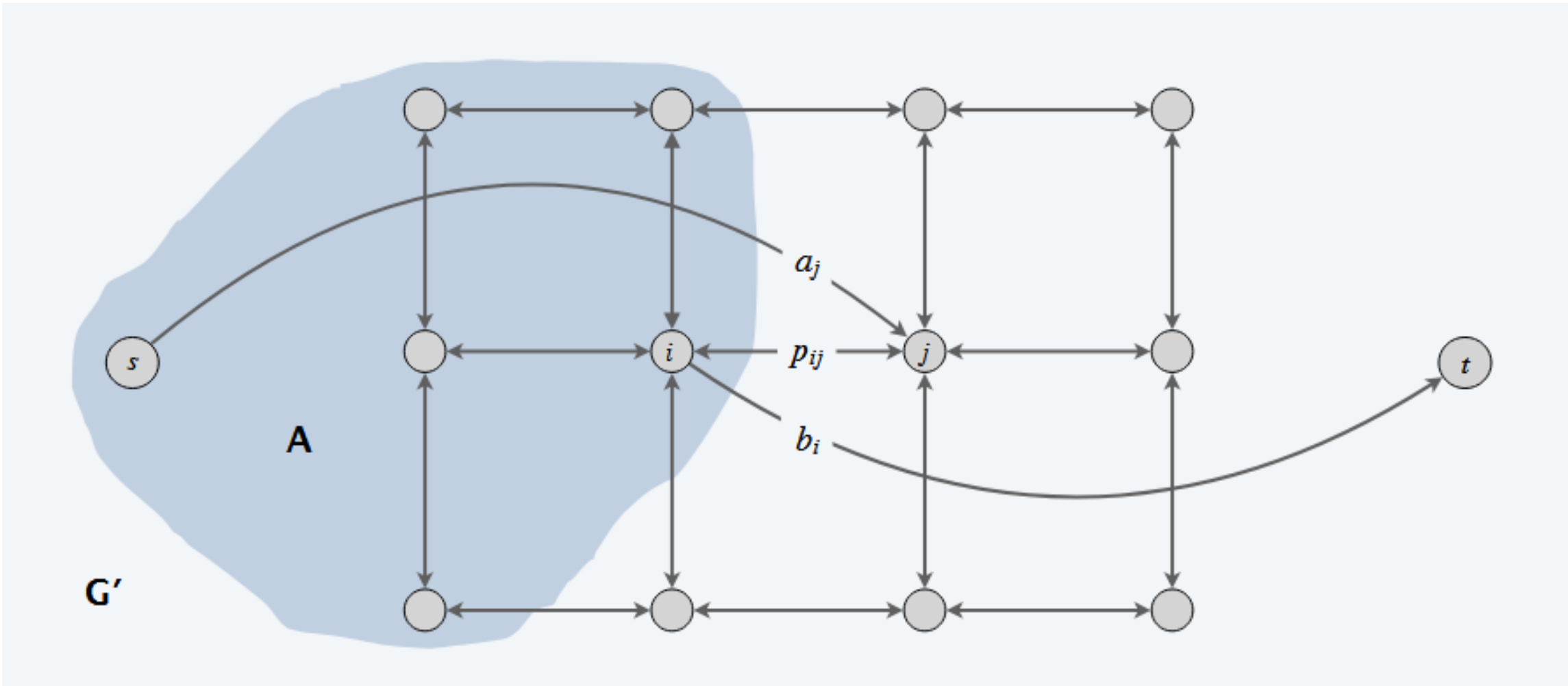
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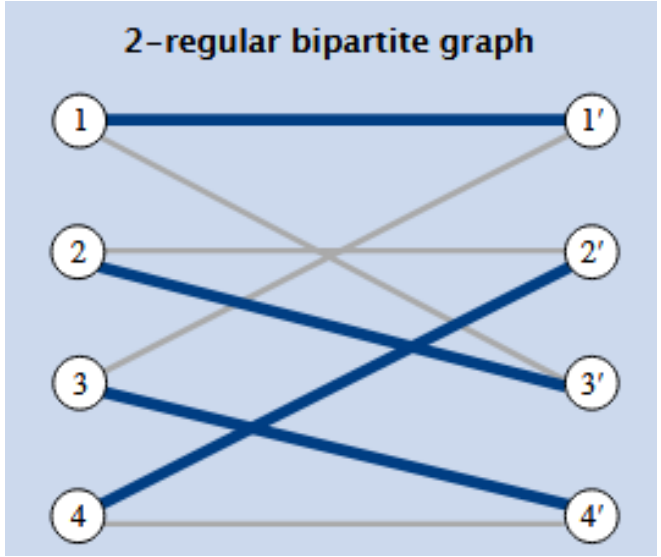
Hackathon problem:

Hackathon attended by n Pomona students and n HMC students.

Each Pomona student is friends with exactly k > 0 HMC students; each Pomona student is friends with exactly k HMC students.

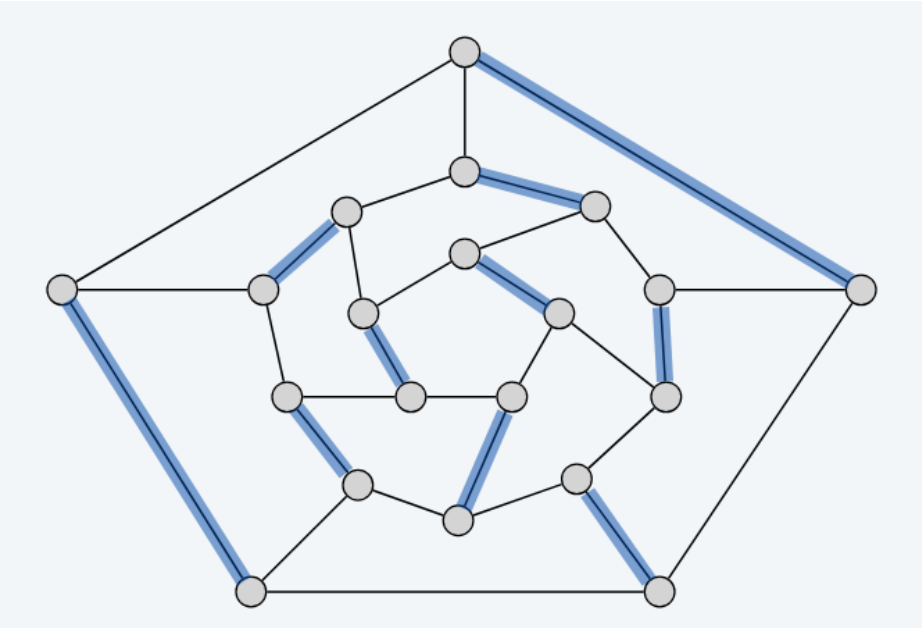
Is it possible to arrange the hackathon so that each Pomona student pair programs with a friend from HMC?

Mathematical reformulation. Is there a perfect matching in this bipartite graph?



Matchings

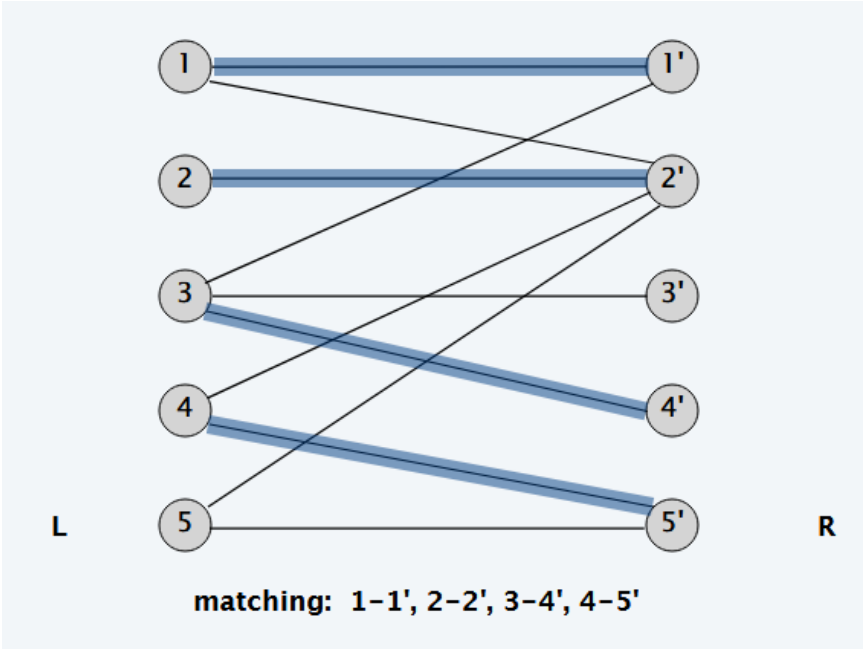
Def. Given an undirected graph G = (V, E), subset of edges $M \subseteq E$ is a matching if each node appears in at most one edge in M.



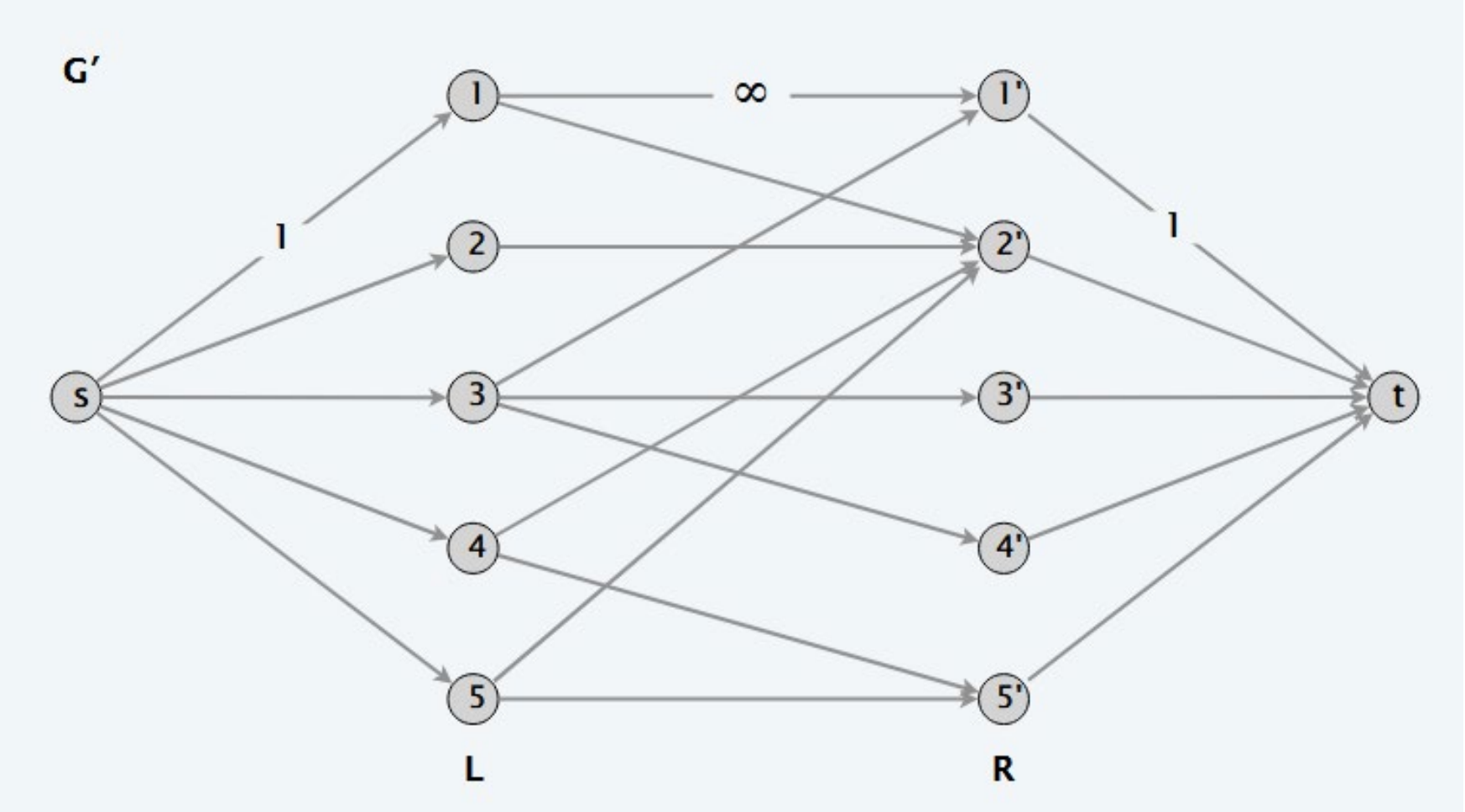
Bipartite Matching

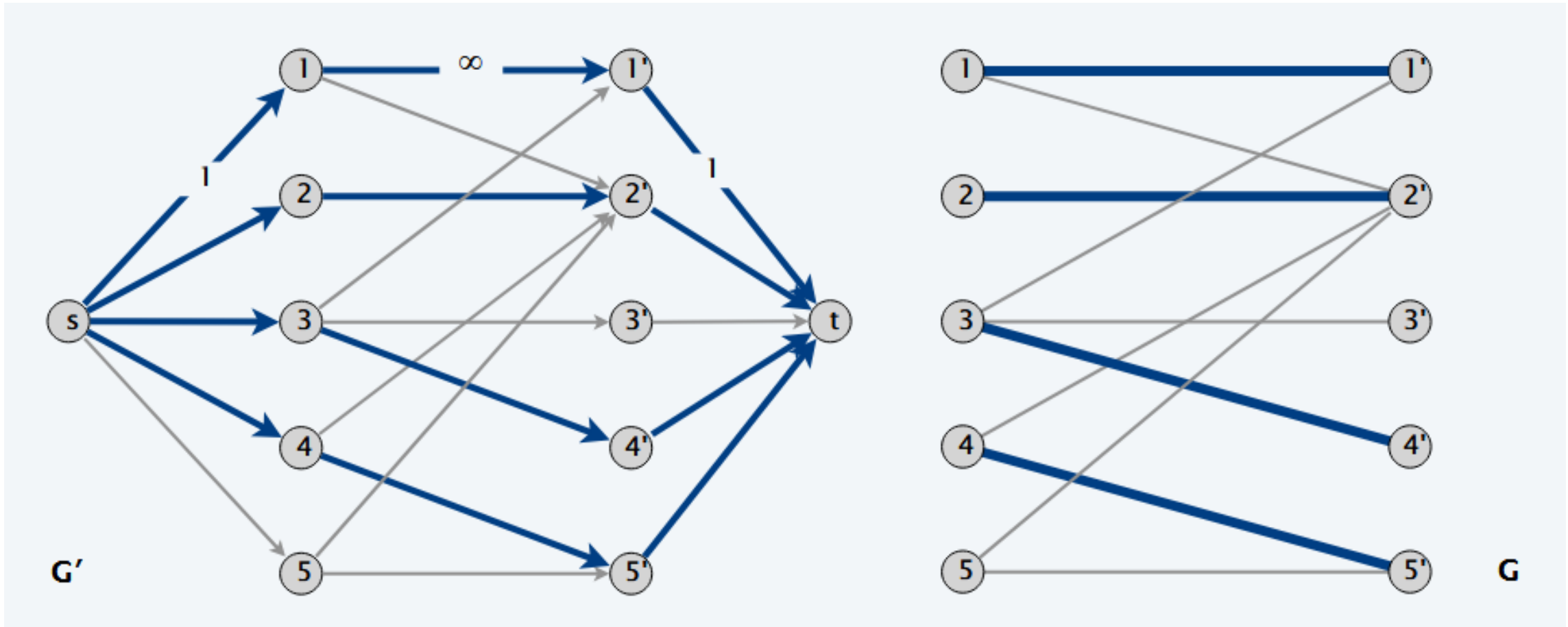
Def. A graph G is bipartite if the nodes can be partitioned into two subsets L and R such that every edge connects a node in L with a node in R.

Bipartite matching. Given a bipartite graph G = (L ∪ R, E), find a max-cardinality matching.



Reduction to flows

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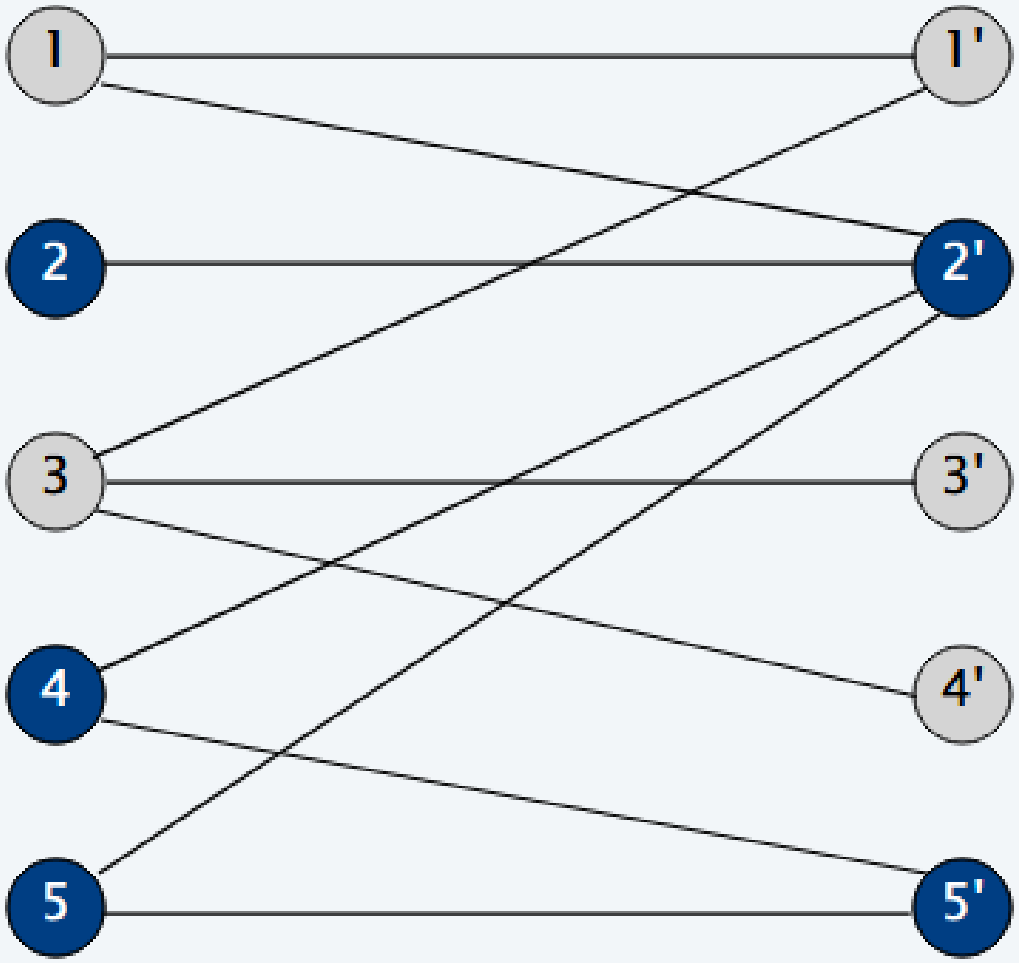
Perfect Matchings in Bipartite Graphs

Q. When does a bipartite graph have a perfect matching?

Structure of bipartite graphs with perfect matchings.

- Clearly, we must have |L| = |R|.
- Which other conditions are necessary?
- Which other conditions are sufficient?



no perfect matching

Hall's Theorem

Theorem: Let $G = (L \cup R, E)$ be a bipartite graph with $|L| = |R|$. Then, graph G has a perfect matching if and only if $|N(S)| \geq |S|$ for all subsets $S \subseteq L$.